OCR
RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4757 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 A mine contains several underground tunnels beneath a hillside. The hillside is a plane, all the tunnels are straight and the width of the tunnels may be neglected. A coordinate system is chosen with the z-axis pointing vertically upwards and the units are metres. Three points on the hillside have coordinates A(15, –60, 20), B(–75, 100, 40) and C(18, 138, 35.6).

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence show that the equation of the hillside is $2x - 2y + 25z = 650$. **[5]**

The tunnel T_A begins at A and goes in the direction of the vector $15\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$; the tunnel T_C begins at C and goes in the direction of the vector $8\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$. Both these tunnels extend a long way into the ground.

(ii) Find the least possible length of a tunnel which connects B to a point in T_A. **[6]**

(iii) Find the least possible length of a tunnel which connects a point in T_A to a point in T_C. **[6]**

(iv) A tunnel starts at B, passes through the point $(18, 138, p)$ vertically below C, and intersects T_A at the point Q. Find the value of p and the coordinates of Q. **[7]**

Option 2: Multi-variable calculus

2 You are given that $g(x, y, z) = x^2 + 2y^2 - z^2 + 2xz + 2yz + 4z - 3$.

(i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. **[3]**

The surface S has equation $g(x, y, z) = 0$, and P(–2, –1, 1) is a point on S.

(ii) Find an equation for the normal line to the surface S at the point P. **[3]**

(iii) A point Q lies on this normal line and is close to P. At Q, $g(x, y, z) = h$, where h is small. Find the constant c such that $PQ \approx c|h|$. **[5]**

(iv) Show that there is no point on S at which the normal line is parallel to the z-axis. **[5]**

(v) Given that $x + y + z = k$ is a tangent plane to the surface S, find the two possible values of k. **[8]**

Option 3: Differential geometry

3 A curve has parametric equations

$$x = a(1 - \cos^3\theta), \quad y = a\sin^3\theta, \quad \text{for } 0 \leqslant \theta \leqslant \frac{\pi}{3},$$

where a is a positive constant.

The arc length from the origin to a general point on the curve is denoted by s, and ψ is the acute angle defined by $\tan\psi = \frac{dy}{dx}$.

(i) Express s and ψ in terms of θ, and hence show that the intrinsic equation of the curve is

$$s = \frac{3}{2}a\sin^2\psi.$$ **[9]**

(ii) For the point on the curve given by $\theta = \frac{\pi}{6}$, find the radius of curvature and the coordinates of the centre of curvature. **[9]**

(iii) Find the area of the curved surface generated when the curve is rotated through 2π radians about the y-axis. **[6]**

Option 4: Groups

4 **(i)** Show that the set $P = \{1, 5, 7, 11\}$, under the binary operation of multiplication modulo 12, is a group. You may assume associativity. **[4]**

A group Q has identity element e. The result of applying the binary operation of Q to elements x and y is written xy, and the inverse of x is written x^{-1}.

(ii) Verify that the inverse of xy is $y^{-1}x^{-1}$. **[2]**

Three elements a, b and c of Q all have order 2, and $ab = c$.

(iii) By considering the inverse of c, or otherwise, show that $ba = c$. **[2]**

(iv) Show that $bc = a$ and $ac = b$. Find cb and ca. **[4]**

(v) Complete the composition table for $R = \{e, a, b, c\}$. Hence show that R is a subgroup of Q and that R is isomorphic to P. **[4]**

The group T of symmetries of a square contains four reflections A, B, C, D, the identity transformation E and three rotations F, G, H. The binary operation is composition of transformations. The composition table for T is given below.

	A	B	C	D	E	F	G	H
A	E	G	H	F	A	D	B	C
B	G	E	F	H	B	C	A	D
C	F	H	E	G	C	A	D	B
D	H	F	G	E	D	B	C	A
E	A	B	C	D	E	F	G	H
F	C	D	B	A	F	G	H	E
G	B	A	D	C	G	H	E	F
H	D	C	A	B	H	E	F	G

(vi) Find the order of each element of T. **[3]**

(vii) List all the proper subgroups of T. **[5]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

A 'random walk' is modelled as a Markov chain with five states A, B, C, D, E representing the possible positions, from left to right, of an object. At each 'step' the object moves as follows.

- If the object is at A, it moves one place to the right (to B).
- If the object is at E, it moves one place to the left (to D).
- Otherwise, the probability that the object moves one place to the left is 0.4, and the probability that it moves one place to the right is 0.6.

Steps occur at intervals of one minute, and the time taken to move may be neglected. The object starts at A, so after the first step (one minute later) the object is at B.

(i) Which of the five states are reflecting barriers? **[1]**

(ii) Write down the transition matrix **P**. **[2]**

(iii) State the possible positions of the object after 10 steps, and give the probabilities that the object is in each of these positions. **[4]**

(iv) Find the probability that after 15 steps the object is in the same position as it was after 13 steps. **[3]**

(v) Find the number of steps after which the probability that the object is at D exceeds 0.69 for the first time. **[3]**

(vi) Find the limits of $\mathbf{P}^{2n}$ and $\mathbf{P}^{2n+1}$ as the positive integer n tends to infinity. **[4]**

(vii) For the interval of 100 minutes between the 200th step and the 300th step, find the expected length of time for which the object is at each of the five positions. **[3]**

(viii) At a certain instant, the object arrives at D. Find the expected number of successive occasions that the object moves to E (and then back to D). Hence find the expected time after this instant when the object first moves to C. **[4]**